MATHEMATICS KANGAROO 2013
Austria - 21.3.2013

Group: Junior, Grades: 9-10

Name:	
School:	
Class:	

Time allowed: 75 min.
Each correct answer, questions 1.-10.: 3 Points
Each correct answer, questions 11.-20.: 4 Points
Each correct answer, questions 21.-30.: 5 Points
Each question with no answer given: 0 Points
Each incorrect answer: Lose ¼ of the points for that question.
You begin with 30 points.

Please write the letter (A, B, C, D, E) of the correct answer under the question number (1 to 30). Write neatly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Information on the Kangaroo contest: www.kaenguru.at
If you want to do more in this area, check out the Austrian Mathematical Olympiad. Info at: www.oemo.at

Ich melde mich zur Teilnahme zum österreichischen Wettbewerb „Känguru der Mathematik 2013“ an.
Ich stimme zu, dass meine personenbezogenen Daten, nämlich Vor- und Zuname, Geschlecht, Klasse, Schulstufe, Schulstandort und Schulart zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von landes- sowie österreichweiten Reihungen, der Veröffentlichung der Ergebnisse jener Schülerinnen und Schüler, die in ihrer Kategorie zumindest 50% der zu vergebenden Punkte erreicht haben sowie des Ermöglichens von Vergleichen mit eigenen Leistungen aus vorherigen Wettbewerbsperioden auf www.kaenguru.at verwendet werden.
Die Verwendung dieser Daten ist bis 31. Dezember 2015 gestattet. Diese Zustimmung kann ich gemäß § 8 Abs. 1 Z 2 DSG 2000 ohne Begründung jederzeit schriftlich bei webmaster@kaenguru.at widerrufen, unter Angabe folgender Informationen zur Identifizierung: Vor- und Zuname des Teilnehmers sowie des Erziehungsberechtigten, der die Zustimmung erteilt hat, Schulstufe und Schule (genaue Adresse), Jahr des Wettbewerbs. Nach dem 31. Dezember 2015 werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei das zuletzt genannte Datum durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art pseudonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage von § 46 Abs. 1 Z 3 DSG 2000 erlaubt.

Unterschrift:

Mathematical Kangaroo 2013
Group Junior (Grade 9./10.)
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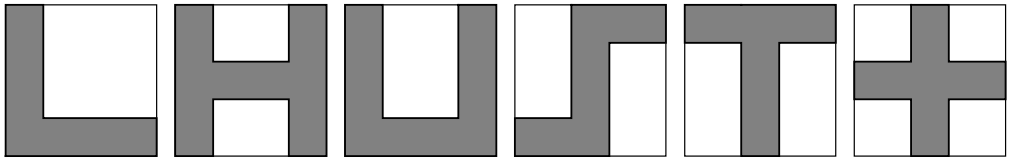
- 3 Point Questions -

1) Which of the numbers is not a factor of 200013 – 2013?

(A) 2 (B) 3 (C) 5 (D) 7 (E) 11

2) Maria has six equally big square pieces of plain paper. On each piece of paper she draws one of the figures shown below. How many of these figures have the same perimeter as the plain piece of paper itself?



(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

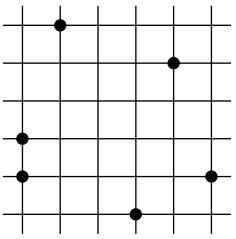
Special corn-on-the-cob offer!
1 Cob 20 Cent
Every 6th cob free!

3) Mrs. Maisl buys four pieces of corn-on-the-cob for each of the four members of her family and get the discount offered. How much does she end up paying?

(A) 0.80 € (B) 1.20 € (C) 2.80 € (D) 3.20 € (E) 80 €

4) The product of three numbers out of the numbers 2, 4, 16, 25, 50, 125 is 1000. How big is the sum of those three numbers?

(A) 70 (B) 77 (C) 131 (D) 143 (E) 177

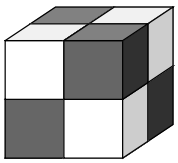


5) On a square grid made up of unit squares, six points are marked as shown on the right. Three of which form a triangle with the least area. How big is this smallest area?

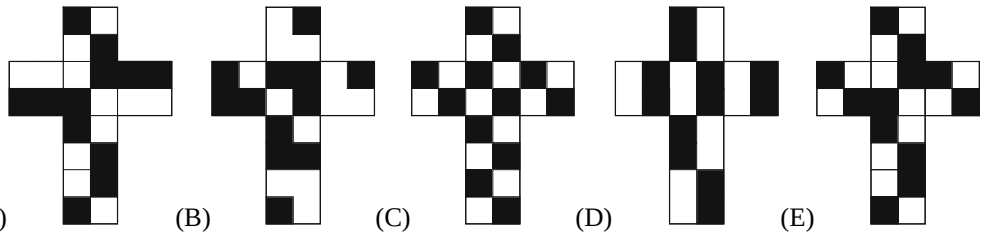
(A) 1/2 (B) 1/3 (C) 1/4 (D) 1 (E) 2

6) If you add 4^{15} and 8^{10}, you obtain a number that is a power of two. Determine that number!

(A) 2^{10} (B) 2^{15} (C) 2^{20} (D) 2^{30} (E) 2^{31}



7) A cube is coloured on the outside as if it was made up of four white and four black cubes where no cubes of the same colour are next to each other (see picture).
Which of the following figures represents a possible net of the coloured cube?

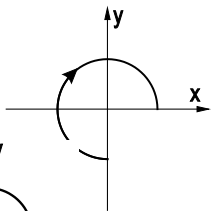


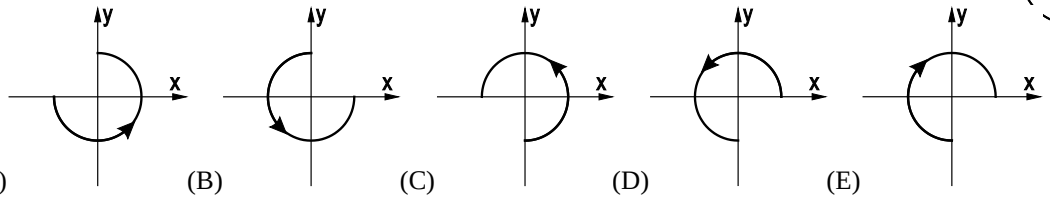
(A) (B) (C) (D) (E)

8) The number n is the biggest natural number for which 4n is three-digits long and m is the smallest natural number for which 4m is three-digits long. Which value does 4n – 4m have?

(A) 900 (B) 899 (C) 896 (D) 225 (E) 224

9) In a drawing we can see a three quarter circle with centre M and an indicated orientation arrow. This three-quarter circle is first turned 90° anti-clockwise about M and then reflected in the x – axis. Which is the resulting picture?



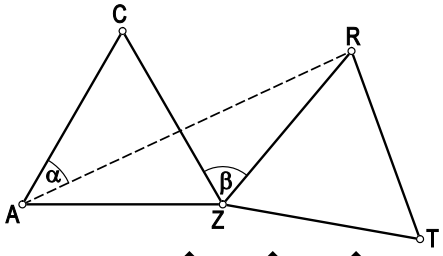


(A) (B) (C) (D) (E)

10) Which of the following numbers is biggest?

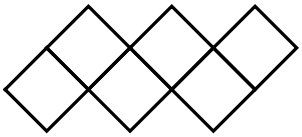
(A) $\sqrt{20} \times \sqrt{13}$ (B) $\sqrt{20} \times 13$ (C) $20 \times \sqrt{13}$ (D) $\sqrt{201} \times 3$ (E) $\sqrt{2013}$

- 4 Point Questions -



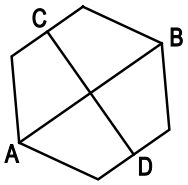
11) Triangle RZT is generated by rotating the equilateral triangle AZC about point Z. Angle $\beta = \angle CZR = 70°$. Determine angle $\alpha = \angle CAR$.

(A) 20° (B) 25° (C) 30° (D) 35° (E) 40°



12) The figure on the right is made up of six unit squares. Its perimeter is 14 cm. Squares will be added to this figure in the same way until it is made up of 2013 unit squares (zigzag: alternating bottom right and top right). How big is the perimeter of the newly created figure?

(A) 2022 (B) 4028 (C) 4032 (D) 6038 (E) 8050

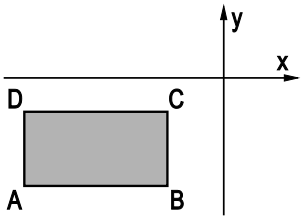


13) A and B are opposite vertices of a regular six-side shape, the points C and D are the mid-points of two opposite sides. The area of the regular six-sided shape is 60. Determine the product of the lengths of the lines AB and CD!

(A) 40 (B) 50 (C) 60 (D) 80 (E) 100

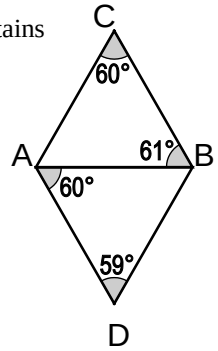
14) A class has written a test. If every boy had obtained 3 more points, the points average would be 1.2 points higher than now. Which percentage of the children in this class are girls?

(A) 20% (B) 30% (C) 40% (D) 60% (E) There is too little information given to determine the answer.



15) The sides of the rectangle ABCD are parallel to the co-ordinate axis. The rectangle lies below the x-axis and to the right of the y-axis, as shown in the diagram. For each of the points A, B, C, D the quotient (y-coordinate):(x- coordinate) is calculated. For which point will you obtain the smallest quotient?

(A) A (B) B (C) C (D) D
(E) It depends on the position of the rectangle and its side lengths.



16) Today is Hans' and his son's birthday. Hans multiplies his age with the age of his son and obtains 2013. In which year was Hans born?

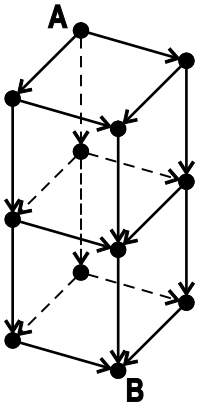
(A) 1952 (B) 1953 (C) 1981 (D) 1982
(E) More information is needed to be able to answer this question.

17) Tarzan wanted to draw a rhombus made up of two equilateral triangles. He drew the line segments inaccurately. When Jane checked the measurements of the four angles shown, she sees that they are not equally big (see diagram). Which of the five line segments in this diagram is the longest?

(A) AD (B) AC (C) AB (D) BC (E) BD

18) Five consecutive positive integers have the following property: The sum of three of the numbers is as big as the sum of the other two. How many sets of 5 such numbers are there?

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3



19) How many different ways are there in the diagram shown, to get from point A to point B if you are only allowed to move in the directions indicated?

(A) 6 (B) 8 (C) 9 (D) 12 (E) 15

20) Given a six-digit number whose digit sum is even and whose digit product is odd. Which of the following statements are true for this number?

(A) Two or four of the digits of this number are even.
(B) There is no such number.
(C) The number of odd digits of this number is odd.
(D) The number can be made up of 6 different digits.
(E) None of the statements (A) – (D) are correct.

- 5 Point Questions -

21) How many decimal places are necessary to write the number $\frac{1}{1024000}$ as a decimal?
(A) 10 (B) 12 (C) 13 (D) 14 (E) 1024000

22) The date 2013 is made up of four consecutive digits 0, 1, 2, 3. How many years before the year 2013 was the date last made up of four consecutive digits?
(A) 467 (B) 527 (C) 581 (D) 693 (E) 990

23) We are looking at rectangles where one side is of length 5.0 cm. Amongst those are some that can be cut into a square and a rectangle one of which has an area of 4,0 cm². How many such rectangles are there?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

24) "Sum change" is a procedure where in a set of three numbers, each number is replaced by the sum of the other two. So for instance {3, 4, 6} becomes the set {10, 9, 7} and this again becomes {16, 17, 19}. Let the starting point be the set {1, 2, 3}.
How many such sum changes are necessary until the number 2013 appears in the set?
(A) 8 (B) 9 (C) 10 (D) 2013 appears several times. (E) 2013 never comes up.

25) Let Q be the number of square numbers amongst the natural numbers from 1 to 2013^6 and K the number of cubic numbers (powers of three) amongst the natural numbers from 1 to 2013^6. Which of the following holds true:
(A) $Q = 2013 \times K$ (B) $2Q = 3K$ (C) $3Q = 2K$ (D) $Q = K$ (E) $Q^3 = K^2$

26) Using the numbers 1, 2, 3, …, 22, 11 fractions $\frac{a}{b}$ are formed where each number is used exactly once. What is the maximum number of fractions with whole number values that can be obtained?
(A) 11 (B) 10 (C) 9 (D) 8 (E) 7

27) Any three vertices of a regular 13-sided-shape are joined up to form a triangle. How many of these triangles contain the circumcentre of the 13-sided-shape?
(A) 72 (B) 85 (C) 91 (D) 100 (E) another number

28) A car starts in point A and drives on a straight road at 50 km/h. Every hour after that another car leaves point A with a speed 1 km/h faster than the one before. The last car leaves A 50 hours after the first car and drives with a speed of 100 km/h. What is the speed of the car that is leading 100 hours after the start of the first car?
(A) 50 km/h (B) 66 km/h (C) 75 km/h (D) 84 km/h (E) 100 km/h

29) 100 trees (oaks and birches) are standing in a row. The number of trees between any two oaks is not equal to 5. What is the maximum number of trees out of the 100 that can be oak trees?
(A) 60 (B) 52 (C) 50 (D) 48 (E) This situation is not possible.

30. A positive integer N is smaller than the sum of its three biggest true factors (N itself is not a true factor of N). Which of the following statements is true?
(A) All such numbers N are divisible by 7.
(B) All such numbers N are divisible by 6.
(C) All such numbers N are divisible by 5.
(D) All such numbers N are divisible by 4.
(E) Such a number N does not exist.