

MATHEMATICS KANGAROO 2012

Austria - 15.3.2012

Group: Junior, Grades: 9-10

Name:	
School:	
Class:	

Time allowed: 75 min.

Each correct answer, questions 1.-10.: 3 Points

Each correct answer, questions 11.-20.: 4 Points

Each correct answer, questions 21.-30.: 5 Points

Each question with no answer given: 0 Points

Each incorrect answer: Lose $\frac{1}{4}$ of the points for that question.

You begin with 30 points.



Please write the letter (A, B, C, D, E) of the correct answer under the question number (1 to 30).

Write neatly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Information on the Kangaroo contest: www.kaenguru.at
 If you want to do more in this area, check out the Austrian Mathematical Olympiad. Info at: www.oemo.at

Ich melde mich zur Teilnahme zum österreichischen Wettbewerb „Känguru der Mathematik 2012“ an.

Ich stimme zu, dass meine personenbezogenen Daten, nämlich Vor- und Zuname, Geschlecht, Klasse, Schulstufe, Schulstandort und Schulart zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von landes- sowie österreichweiten Reihungen, der Veröffentlichung der Ergebnisse jener Schülerinnen und Schüler, die in ihrer Kategorie zumindest 50% der zu vergebenden Punkte erreicht haben sowie des Ermöglichens von Vergleichen mit eigenen Leistungen aus vorherigen Wettbewerbsperioden auf www.kaenguru.at bzw. <http://kaenguru.diefenbach.at/> verwendet werden.

Die Verwendung dieser Daten ist bis 31. Dezember 2014 gestattet. Diese Zustimmung kann ich gemäß § 8 Abs. 1 Z 2 DSGVO 2000 ohne Begründung jederzeit schriftlich bei webmaster@kaenguru.at widerrufen. Nach dem 31. Dezember 2014 werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei das zuletzt genannte Datum durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art pseudonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage von § 46 Abs. 1 Z 3 DSGVO 2000 erlaubt.

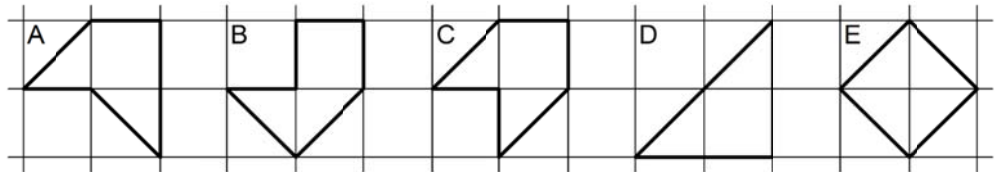
Unterschrift:

Mathematics Kangaroo 2012 Group Junior (Grades 9./10.) Austria - 15.3.2012



- 3 Point Questions -

1. Which of the shapes to the right has the largest area?



- (A) A (B) B (C) C (D) D (E) All shapes have the same area.

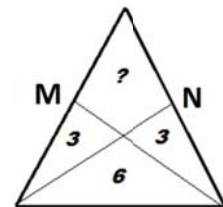
2. $11.1111 - 1.1111 =$

- (A) 9.0009 (B) 9.0909 (C) 9.9999 (D) 9.999 (E) 10

3. A wristwatch was laid on a table in such a way that the minute hand pointed northeast. How many minutes must pass before the minute hand is pointing northwest for the first time?

- (A) 45 (B) 40 (C) 30 (D) 20 (E) 15

4. M and N are the midpoints of the equal sides of an isosceles triangle. How big is the area of the quadrilateral (marked ?)?

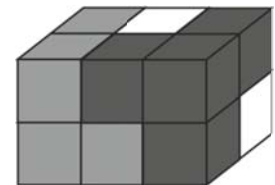
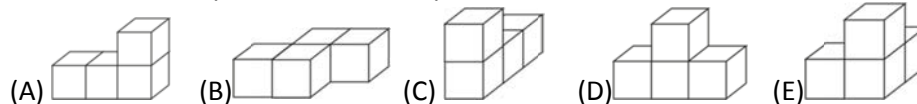


- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

5. To the number 6 we add 3. We multiply the result with 2 and add 1. What is the result of this calculation?

- (A) 27 (B) 7 (C) 18 (D) 20 (E) 19

6. A cuboid is formed from 3 pieces (see picture). Each piece is made from 4 cubes of the same colour. What shape does the white piece have?



7. The digit sum of a seven digit number is 6. What is the product of the digits?

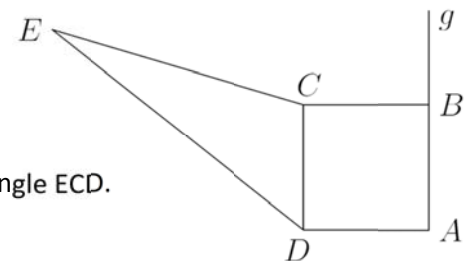
- (A) 0 (B) 6 (C) 7 (D) $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$ (E) 5

8. ABC is a right-angled triangle with shorter sides 6cm and 8cm. K, L, M are the midpoints of the sides of triangle ABC. What is the perimeter of triangle KLM?

- (A) 10 cm (B) 12 cm (C) 15 cm (D) 20 cm (E) 24 cm

9. The quadrilateral ABCD with side length 4cm has the same area as triangle ECD. What is the perpendicular distance from point E to the line g?

- (A) 8 cm (B) $(4 + 2\sqrt{3})$ cm (C) 12 cm
(D) $10 \times \sqrt{2}$ cm (E) It depends on the position of E.



10. Alice and Bob send each other secret messages. To put their messages into code they use the following system: First each letter is given a number in order: A = 1, B = 2, C = 3, ... Z = 26. Then the letter number is doubled and 9 is added. Bob received a message which began 19 – 37 – 48 – 19 – ... Which of the following messages had Alice sent to Bob?

- (A) ENDE... (B) ENTE... (C) ERNA... (D) ERDE... (E) Alice has made a mistake

- 4 Point Questions -

11. In four of the following calculations you can swap the number 8 with another positive number without changing the answer to the sum. For which calculation does it not work?

- (A) $(8 + 8 - 8) \div 8$ (B) $8 + (8 \div 8) - 8$ (C) $8 \div (8 + 8 + 8)$
 (D) $8 - (8 \div 8) + 8$ (E) $8 \times (8 \div 8) \div 8$

12. When Adam stands on a table and Mike on the floor, Adam is 80cm taller than Mike. When Mike stands on the table and Adam on the floor, Mike is one metre taller than Adam. How high is the table?

- (A) 20 cm (B) 80 cm (C) 90cm (D) 100 cm (E) 120 cm

13. When the numbers 144 and 220 are divided by the same positive whole number x , both have remainder 11. Find x .

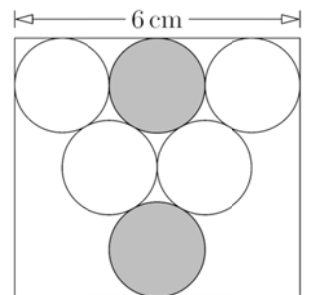
- (A) 7 (B) 11 (C) 15 (D) 19 (E) 38

14. Tom and Mary play a game with a coin. When the coin shows heads, Mary wins and Tom must give her two sweets. When the coin shows tails Tom wins and Mary must give him three sweets. After 30 throws of the coin they each have the same number of sweets as they had at the start of the game. How often has Tom won?

- (A) 6 (B) 12 (C) 18 (D) 24 (E) 30

15. One of the two sides of a rectangle has length 6cm. In the rectangle circles are drawn next to each other in such a way that their centres form an equilateral triangle. What is the shortest distance between the two grey circles (in cm)?

- (A) 1 (B) $\sqrt{2}$ (C) $2\sqrt{3} - 2$ (D) $\frac{\pi}{2}$ (E) 2

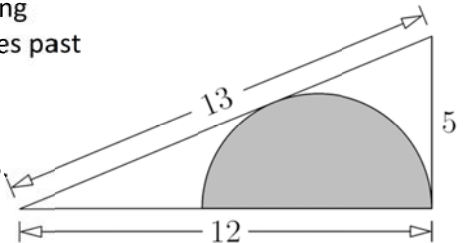


16. On each of the four walls in Billy's room hangs a correctly working clock, but each one runs either behind or ahead of the correct time. The first clock is incorrect by 2 minutes, the second by 3 minutes, the third by 4 minutes and the fourth by 5 minutes. Billy wants to know what time it is and sees the following times; 6 minutes to 3, 3 minutes to three, 2 minutes past three and 3 minutes past 3. What is the actual time?

- (A) 3:00 (B) 2:57 (C) 2:58 (D) 2:59 (E) 3:01

17. The diagram shows a right-angled triangle with side lengths 5, 12 and 13. What is the length of the radius of the inscribed semi-circle?

- (A) $7/3$ (B) $10/3$ (C) $12/3$ (D) $13/3$ (E) $17/3$



18. How many numbers from 1000 to 9999 are there which have 3 as the hundred digit, and for which the sum of the remaining three digits is also 3?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

19. A number from 1 to 9 is to be written into each of the 12 fields of the table so that the sum of each column is the same. Also the sum of each row must be the same. A few numbers have already been written in. Which number should be written in the grey square?

- (A) 1 (B) 4 (C) 6 (D) 8 (E) 9

2	4		2
	3	3	
6		1	

20. The runners Kann, Gu and Ru are favourites to win the marathon. Before the race three experts gave their predictions for the outcome of the race.

Expert 1: "Either Kann or Gu will win."

Expert 2: "If Gu is second Ru will win."

Expert 3: "If Gu is third Kan will not win."

Expert 4: "Either Gu or Ru will come second."

After the race all four predictions were proven correct. In which order, did the three runners finish the race?

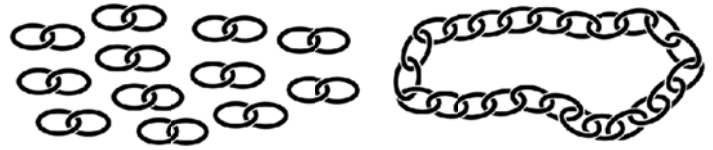
- (A) Kan, Gu, Ru (B) Kan, Ru, Gu (C) Ru, Gu, Kan (D) Gu, Ru, Kan (E) Gu, Kan, Ru

- 5 Point Questions -

21. Two sides of a quadrilateral have lengths 1 and 4. One of the diagonals has length 2 and splits the quadrilateral into two isosceles triangles. What is the perimeter of the quadrilateral?

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

22. A goldsmith has 12 double-links of chain. Out of these he wants to make a single closed chain with 24 links. What is the minimum number of links that he must open (and close again)?



- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

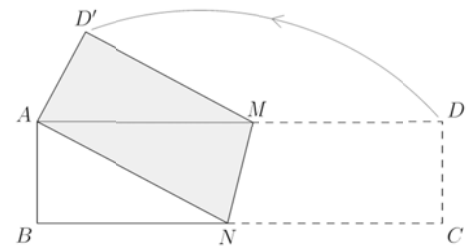
23. Peter wrote the number 2012 in the form $2012 = m^m(m^k - k)$ where m and k are natural numbers. Find the value of k .

- (A) 2 (B) 3 (C) 4 (D) 9 (E) 11

24. What is the last non zero digit of $K = 2^{59} \times 3^4 \times 5^{53}$?

- (A) 1 (B) 2 (C) 4 (D) 6 (E) 9

25. A rectangle ABCD with dimensions 16cm by 4cm was folded along the line MN so that corner C meets corner A. What is the area of the Pentagon ABNMD'?

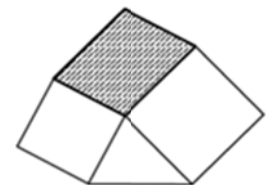


- (A) 17 cm² (B) 27 cm² (C) 37 cm² (D) 47 cm² (E) 57 cm²

26. It takes 8 seconds for train G to pass by a milestone. Shortly afterwards the train meets train H. It takes 9 seconds for the trains to pass each other. Train H then takes 12 seconds to pass by the milestone. What can be deduced about the length of the trains?

- (A) G is twice as long as H. (B) They are the same length. (C) H is 50% longer than G
(D) H is twice as long as G. (E) Nothing can be deduced.

27. The shape pictured, is made out of two squares with side lengths 4cm and 5cm respectively, a triangle with area 8 cm² and the grey parallelogram. What is the area of the parallelogram?



- (A) 15 cm² (B) 16 cm² (C) 18 cm² (D) 20 cm² (E) 21 cm²

28. Of 5 lamps each one can be set to "ON" or "OFF". Each time when the switch of one lamp is changed, not only does the status of that particular lamp change but also that of one other lamp chosen at random. If the same switch is changed several times not always the same other lamp changes. Initially all lamps are set to "OFF". Then 10 switching operations are carried out. After that one can say that

- (A) definitely not all lamps are switched to "OFF";
(B) definitely all lamps are switched to "ON";
(C) definitely not all lamps are switched to "ON";
(D) definitely all lamps are switched to "OFF";
(E) none of the statements in (A) to (D) are true.

29. The natural numbers from 1 to 120 were written as shown into a table with 15 columns. In which column (counting from left) is the sum of the numbers the largest?

1							...	
2	3						...	
4	5	6					...	
7	8	9	10				...	
11	12	13	14	15			...	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
106	107	108	109	110	111	112	...	120

- (A) 1 (B) 5 (C) 7 (D) 10 (E) 13

30. Positive numbers were written in a 3 × 3 grid in such a way that the product of the numbers in each row and each column is exactly 1. The product of the four numbers in each 2 × 2 grid that can be found inside the 3 × 3 grid is 2. Which number is written in the centre of the 3 × 3 grid?

- (A) 16 (B) 8 (C) 4 (D) $\frac{1}{4}$ (E) $\frac{1}{8}$