

**The Mathematics Kangaroo 2004**  
**Students (grades 11 or 12)**  
**18.3.2004**



- 3-point questions -

1) If I buy  $m$  pens for  $n$  Euro each, and  $n$  pens for  $m$  Euro each, what will be the average price for each pen?

- A) 1      B)  $\frac{m+n}{2}$       C)  $\frac{2mn}{m+n}$       D)  $mn$       E)  $\frac{m^2n^2}{2}$

2) Peter has 2004 marbles. Half of them are blue, a quarter of them are red and one sixth of them are green. How many marbles have a color other than blue, red or green?

- A) 167      B) 334      C) 501      D) 1002      E) 1837

3) A house has a rectangular base of dimension 40 m x 60 m. On an architect's drawing the perimeter of the base is 100 cm. What scale was used for the drawing?

- A) 1 : 100      B) 1 : 150      C) 1 : 160      D) 1 : 170      E) 1 : 200

4) A pyramid has 16 faces. How many edges does it have?

- A) 16      B) 17      C) 18      D) 32      E) 34

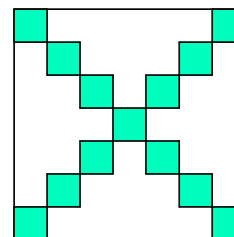
5) The smallest real number  $x$ , for which  $x^2 - 2004 \leq 0$ , is

- A) -2004      B) 2004      C) 0      D)  $\sqrt{2004}$       E)  $-\sqrt{2004}$

6) Each inhabitant of planet Mars has either one, two or three tentacles on their head. Exactly 1% of the population have three 97% have two and 2% have one. How many percent of the population have more tentacles than is the average of the entire population?

- A) 1%      B) 3%      C) 97%      D) 98%      E) 99%

7) A number  $s$  is an odd number. Along the diagonals of a square of side  $s$  there are small unit squares, as indicated in the square on the right whose sides measure 7 units. How big is the remaining white area?



- A)  $s^2+1-2s$       B)  $s^2+4-4s$       C)  $2s^2+1-4s$       D)  $s^2-1-2s$       E)  $s^2-2s$

8) The squares and cubes of how many two-digit numbers have the same unit digit?

- A) 1      B) 9      C) 10      D) 21      E) more than 30

9) A large square is made up of 18 small squares, of which 17 have sides of measure 1. The area of the large square is therefore

- A) 25      B) 49      C) 81      D) 100      E) 225

10) How many right triangles can be formed by connecting vertices of a regular 14-gon?

- A) 72      B) 82      C) 84      D) 88      E) another number

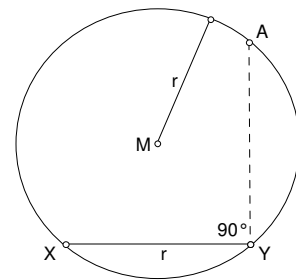
**- 4-point questions-**

11) 15 sheep and a certain number of shepherds are on a field. After half the shepherds and one third of the sheep left 50 legs remain on the field. How many legs were on the field at the beginning?

- A) 60      B) 72      C) 80      D) 90      E) 100

12) Which angle is included by the line segments AX and AY?

- A)  $22\frac{1}{2}^\circ$       B)  $30^\circ$       C)  $45^\circ$       D)  $60^\circ$       E)  $90^\circ$



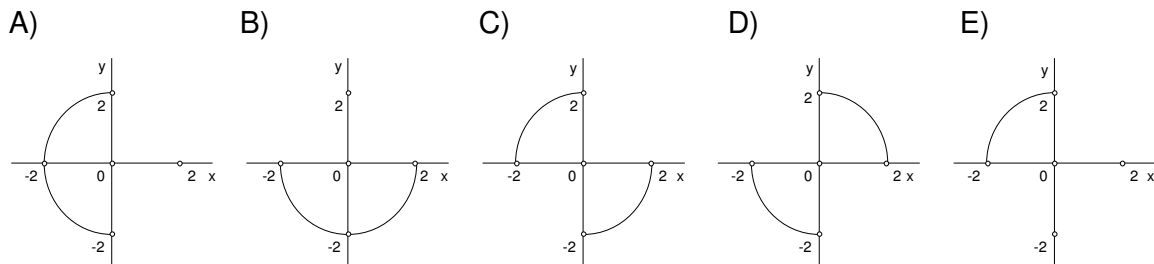
13) How many squares have one vertex  $A(-1/-1)$  and are symmetric with respect to either coordinate axis?

- A) 2      B) 3      C) 4      D) 5      E) 6

14) 100 spheres, numbered from 1 to 100 are put into an opaque tumbler. How many spheres have to be taken out of the tumbler, in order to ensure that the product of the numbers on the spheres is divisible by 4?

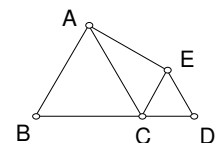
- A) 51      B) 52      C) 53      D) 54      E) 55

15) Which of the graphs below represents the set of points with coordinates  $(x, y)$  such that  $xy < 0$  and  $|x|^2 + |y|^2 = 4$ ?



16) In the drawing on the right ABC and ECD are equilateral triangles with sides of length 2 and 1 respectively. The area of the quadrilateral ABCE is

- A)  $\frac{5\sqrt{3}}{3}$       B)  $\frac{4+5\sqrt{3}}{4}$       C) 3      D)  $\frac{6+\sqrt{3}}{4}$       E)  $\frac{3\sqrt{3}}{2}$



17) How many positive integers can be written as  $a_0 + a_1 3 + a_2 3^2 + a_3 3^3 + a_4 3^4$  if  $a_0, a_1, a_2, a_3, a_4$  are elements of  $\{-1, 0, 1\}$ ?

- A) 5      B) 80      C) 81      D) 121 E) 243

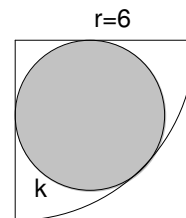
18) The number  $(\sqrt{22 + 12\sqrt{2}} - \sqrt{22 - 12\sqrt{2}})^2$  is

- A) negative      B) equal to 0      C) the 4th power of a positive integer  
D) equal to  $11\sqrt{2}$       E) a positive integer that is divisible by 5

19) The sum of the interior angles of a regular polygon is one-seventh the sum of the interior angles of a regular 16-gon. How many vertices does the polygon have?

- A) 3      B) 4      C) 6      D) 7      E) 10

20) A circle  $k$  is inscribed into a quarter circle of radius 6 as shown on the right. What is the radius of  $k$ ?



- A)  $\frac{6-\sqrt{2}}{2}$       B)  $\frac{3\sqrt{2}}{2}$       C) 2,5      D) 3      E)  $6(\sqrt{2}-1)$

**- 5-point questions-**

21) (A geometric sequence is a sequence for which a number  $q$  exists, such that  $a_{n+1} = q a_n$ , for all natural numbers  $n$ ). If for a geometric sequence  $(a_n)_{n \geq 1}$  it is true that  $a_3 < a_2 < a_4$ . then

- A)  $a_3 \cdot a_4 > 0$       B)  $a_2 \cdot a_3 < 0$       C)  $a_2 \cdot a_4 < 0$       D)  $a_2 < 0$       E)  $a_2 \cdot a_3 > 0$

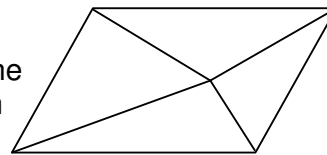
22) Which is the second last digit of  $11^{2004}$ ?

- A) 0      B) 1      C) 2      D) 3      E) 4

23) The Broccoli-party in Gemm-city is running for elections first time. All who have voted for it have eaten broccoli once before, while 90% of those who voted for any other party never have eaten broccoli. What percentage of voters voted for the Broccoli-party, if 46% of all voters have eaten broccoli once before?

- A) 40%      B) 41%      C) 43%      D) 45%      E) 46%

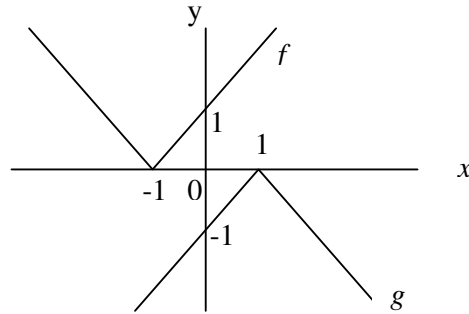
24) A parallelogram is divided into 4 triangles (as shown). Of the following sequences of 4 numbers at most one of them represents the areas of the triangles in some kind of sequence. Which one is possible?



- A) 14,15,18,19      B) 8,9,10,15      C) 10,11,12,19      D) 11,13,15,16  
E) none of the four are possible.

25) On the right you see the graphs of two real functions  $f$  and  $g$ . What is the relation between  $f$  and  $g$  for all real values of  $x$ ?

- A)  $f(x) = -g(x) + 2$       B)  $f(x) = -g(x) - 2$   
 C)  $f(x) = -g(x+2)$       D)  $f(x+2) = -g(x)$   
 E)  $f(x+1) = -g(x-1)$



26) ABC is an equilateral triangle of side 4. What is the radius of the arc with center A, that divides the area of the triangle into two equal parts?

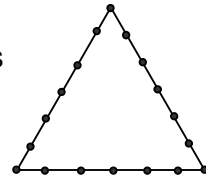
- A)  $\sqrt{\frac{12\sqrt{3}}{\pi}}$     B)  $\sqrt{\frac{24\sqrt{3}}{\pi}}$     C)  $\sqrt{\frac{30\sqrt{3}}{\pi}}$     D)  $\frac{6\sqrt{3}}{\pi}$     E)  $\sqrt{\frac{48\sqrt{3}}{\pi}}$

27) Given 200 numbers. At first they are all zeros. As a first step 1 is added to each number. As a second step 1 is added to every second number starting from the very left. As a third step 1 is added to every third number, and so on. Which number will be on place 120 after 200 steps?

- A) 16      B) 12      C) 32      D) 24      E) 20

28) How many triangles have their (non collinear) vertices in the points shown in the figure on the right?

- A) 816      B) 711      C) 777      D) 717      E) 811



29) The sum of the 3 digits of all 3 digit numbers with different digits, that can be formed with the digits  $0 < a < b < c$ , is 1554. What is the value of  $c$ ?

- A) 3      B) 4      C) 5      D) 6      E) 7

30) The number  $m = 999\dots 9$  is written with 999 digits of 9. What is the sum of the digits of  $m^2$ ?

- A) 8982      B) 8991      C) 9000      D) 9009      E) 9018